

Relation between Transmission Group Delay and Stored Energy in Optical Fibers

Z. Várallyay, P. G. Antal, R. Szipőcs*

Institute for Solid State Physics and Optics, Wigner RCP of HAS, H-1121 Budapest, Konkoly Thege str. 29-33, HUNGARY

**Author e-mail address: szipocs.robert@wigner.mta.hu*

Abstract: We show that the group delay of an optical pulse of finite bandwidth transmitted through a piece of optical fiber of unit length is proportional to the energy stored by the standing wave electromagnetic field.

I. INTRODUCTION

Dispersive properties of optical fibers play an important role in long distance, high speed optical data transmission systems [1] and in ultrashort (ps or fs) pulse optical fiber laser systems [2]. Recently we showed [3] that reversed or flat dispersion functions in a wide wavelength range can be obtained by introducing resonant structures in the fiber cladding in hollow-core, air-silica photonic bandgap fibers or in solid core Bragg fibers with step-index profile.

In this paper we discuss the physics behind the operation of “dispersive” optical fibers, i.e., optical fibers designed for dispersion (D) or dispersion slope (S) compensation to the second- or third-order. In general, we show that the group delay (τ) of a relatively narrowband optical pulse transmitted through a piece of optical fiber of unit length is proportional to the energy stored by the standing wave electromagnetic field at the same (central) frequency, as far as the confinement loss is small. This strong relationship between these two physical quantities is not surprising at all, but has not been emphasized and used for the design of “dispersive” optical fibers. We are convinced that having this relationship in mind one can construct higher performance “dispersive” optical fibers, such as high-order mode (HOM) fibers [4], and hollow- or solid-core photonic bandgap (PBG) fibers [3].

II. THEORY

The group delays in a dielectric structure can be related to the stored energy using the Poynting's theorem [5]

$$-\oint_S \mathbf{S} \cdot \hat{n} d\mathbf{a} = \frac{dU}{dt} \quad (1)$$

where U is the electromagnetic field energy and $\mathbf{S}$ is the Poynting vector. Evaluating the surface integral in (1) yields the ratio between the energy and group-delay as shown in Ref. [5,6]. Also, numerical simulations in 1D PBG structures was carried out indicating that the reflection group delay is approximately proportional to the stored energy if the incident power is constant: $\tau_{\text{gr}} \propto U$ [6].

We check this theory for both photonic bandgap and index guiding fibers by a few numerical examples. We derive the dispersion as well as the energy in the fiber structures at those wavelength regions where the loss of the fiber is relatively small in order to stay close to the unit reflection from the cladding.

For calculation of the group delay for transmission over the unit length of the fiber, we obtain the effective refractive index (n_{eff}) as the eigenvalue of the Helmholtz equation by evaluating the LP_{01} mode at different wavelengths in the optical fibers.

For our studies, we calculate the transmission group delay for a piece of optical fiber as follows

$$\tau = \frac{d\phi}{d\omega} = \frac{d}{d\omega} \left[\frac{\omega L}{c} n_{\text{eff}}(\omega) \right] \quad (2)$$

where $n_{\text{eff}}(\omega)$ and L are the frequency dependent effective refractive index and the length of the fiber, respectively.

The stored energy (U) in a piece of the fiber with length L can be derived from the electric and magnetic field distributions (eigenfunctions) we obtain from the Helmholtz eigenvalue equation as follows:

$$U = \iiint u dV = \iiint \left[\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right] dV = \frac{L}{2} \iint \left[\epsilon_0 n^2 E^2 + \mu_0 H^2 \right] dA \quad (3)$$

where the simplification in the integral results from the longitudinal uniformity (E and H are independent of the z coordinate) and $\mu_r = 1$. The normalization of the modes at different frequencies were done by choosing

$$\frac{1}{2} \iint [\vec{E} \times \vec{H}^*] \cdot \vec{u}_z dA = 1 \quad (4)$$

where $\vec{u}_z$ is the unit vector in the axial (z) direction. Note that this normalization procedure assures that the incident power (P_i) is unity for the different frequencies.

III. NUMERICAL RESULTS

We investigate a standard, low confinement loss single mode and index guiding fiber type HI-1060 of Corning. In our simulations, we used refractive index data of fused silica for the cladding by using the corresponding Sellmeier formula, while the refractive index difference between the core and the cladding was kept constant $\Delta n = 0.0075$ over the 920 and 1200 nm range. The core diameter is $d_{\text{core}} = 5.3 \mu\text{m}$. In order to check the effect of a finite, frequency dependent, non-zero confinement loss in optical fibers, we performed similar simulations for solid core photonic bandgap fibers of two different kinds. The low index layers in the investigated fiber structures are made of fused silica, while the refractive indices of the high index layers are higher by 0.015 than that of the low index layer at each wavelength. The core radius of the fiber is $R_c = 8 \mu\text{m}$ and the thicknesses of the alternating high and low index layers are $d_H = 1.4 \mu\text{m}$ and $d_L = 4.9 \mu\text{m}$, respectively. The number of HL periods in the cladding region is seven. One of the designs has a standard quarterwave-stack structure, while the another has a resonant layer around the core in order to minimize the dispersion slope at around 1 micron [3]. The resonant layer has refractive index difference 0.008 relative to fused silica and a physical thickness of $1.6 \mu\text{m}$.

The computed dispersions and confinement losses are shown in Fig. 1(a), while the ratio between the stored energy and the group delay is depicted in Fig. 1(b) for the different fibers. It is clearly seen that the ratio is very close to unity all over the fiber transmission band, furthermore, the lower confinement loss increases the energy-group delay ratio.

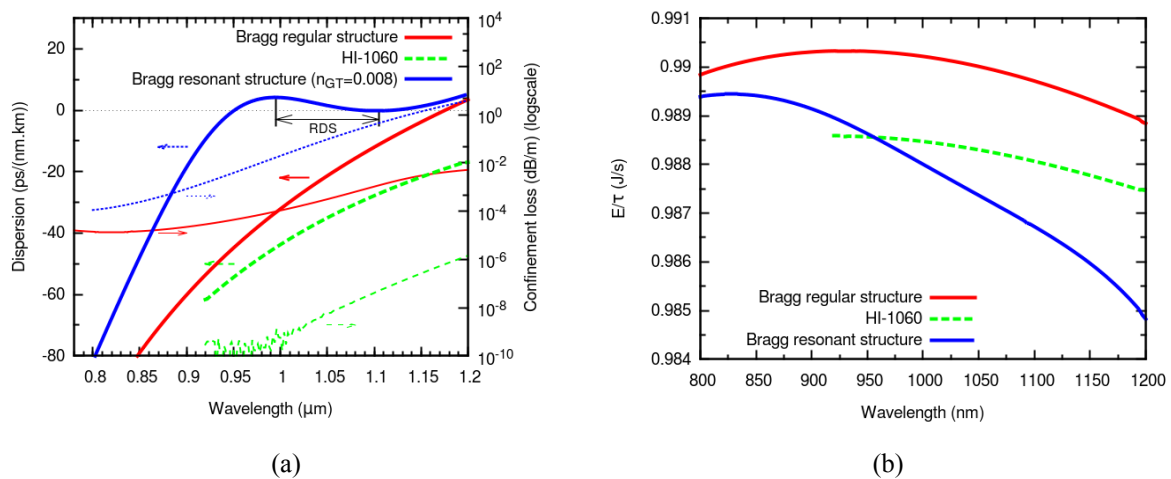


Fig. 1.(a) Computed dispersions (thick lines) and confinement losses (thin lines) and (b) the computed stored energy-group delay ratio in solid core PBG fibers of two different designs having a unit length of 1 m (incident power is 1W) and for HI-1060 index-guiding fiber.

IV CONCLUSION

We found that the U/τ ratio is very close to unity all over the fiber transmission band, furthermore, the lower confinement loss increases the energy-group delay ratio close to 1. Since the stored energy in the fiber is derived from the electric field distributions at each wavelength (see Eq. (3)), we can say that modifications in the dispersion function of any optical fiber always result in a change in the mode field distribution at each wavelength, and vice versa. From this relationship, it also follows that surface modes appearing in hollow core PBG fibers seriously affect the dispersion profile of these optical fibers [3]. This relationship in mind one can construct “dispersive” optical fibers, such as HOM fibers [4], and hollow- or solid-core photonic bandgap (PBG) fibers [3] having the necessary group-delay dispersion but being aware of the mode field distortions.

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