Relation between group delay and energy storage in dispersive dielectric mirror coatings

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Abstract. We show that the reflection group delay of a highly reflective, dielectric multilayer mirror is proportional to the energy stored by the standing wave electromagnetic field built up in such 1D photonic bandgap (PBG) structures.

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1. Introduction

Phase properties of dielectric multilayer mirrors have attracted great interest since the first chirped mirror (CM) coating was invented [1]. We developed these mirrors for intra- or extra-cavity, broadband feedback and dispersion control in femtosecond (fs) pulse solid state laser systems. Dispersion of a chirped mirror structure primarily originates from the frequency dependent penetration depth of the different spectral components. Dispersion compensation by laser mirrors may also rely on resonances built up in the multilayer structure, like in the case of a Gires-Tournois interferometer (GTI) mirror [2], where a resonance caused by a tiny GTI cavity is responsible for the negative group delay dispersion over a limited bandwidth (typically 1-5% of the central frequency). In contrast to CM-s, the maximum value of group delay on reflection is not limited by the overall optical thickness of the GTI type multilayer coating, hence higher values of the group delay dispersion (GDD) can be obtained over a higher bandwidth (typically 5-15% of the central frequency) with dispersive mirror designs comprising properly tailored, multiple cavities (MCGTI mirrors) [3,4]. Both in chirped mirror structures and GTI-type mirrors, a standing wave field builds up for each spectral components inside the multilayer structure from the incident electromagnetic wave. In this paper we show that energy stored by the standing wave field determines the phase properties of the dielectric multilayer structures. This relationship might lead to a better understanding of dispersive properties of multilayer mirror coatings as well as to new design approaches in 1D or multi-dimensional dispersive PBG structures.

2. Theory

Let E_i be the incident electric field in the frequency domain. The transmitted and reflected electric fields can be respectively written as

$$E_t(\omega) = E_i \left| t(\omega) \right| \exp\left(i\phi_t(\omega) \right), \tag{1}$$

$$E_r(\omega) = E_i \left| r(\omega) \right| \exp\left(i\phi_r(\omega) \right), \tag{2}$$

where ω is the angular frequency, $t(\omega)$ and $r(\omega)$ are the complex amplitude transmission and reflection coefficients, respectively, $\phi_t(\omega)$ and $\phi_r(\omega)$ are the arguments of these coefficients. The group delay on reflection and on transmission are the first derivatives of these phases:

$$\tau_{gr} = \frac{d\phi_r(\omega)}{d\omega}, \qquad \tau_{gt} = \frac{d\phi_t(\omega)}{d\omega}. \tag{3}$$

The stored energy density can be written as

$$u(r,t) = \frac{1}{2}\left(n^2 \varepsilon_0 E^2 + \mu_0 H^2 \right). \tag{4}$$

The whole stored electromagnetic energy (U) in the multilayer is the volume integral of this energy density over the volume of the structure. Winful has shown that the so called dwell time (τ_d) is proportional to the stored electromagnetic energy [5]:

$$\tau_d \equiv |t|^2 \tau_{gt} + |r|^2 \tau_{gr} = \frac{U}{P_i} \tag{5}$$

where P_i is the incident power. For a nonabsorptive medium, the sum of the intensity reflection and transmission coefficients is one: $|r|^2 + |t|^2 = 1$. Therefore if the reflection is close to unity, the transmittance is close to zero. It is apparent from (5) that if the reflection is high, the reflection group delay is proportional to the stored energy:

$$|r|^2 \approx 1 \Rightarrow \tau_{gr} \approx \frac{U}{P_i}. \tag{6}$$

So if the incident power spectral density is independent from the wavelength, the ratio U/τ_{gr} should be constant in the spectral region where the reflection is high.

3. Numerical results

We examined four different highly reflective (HR) multilayer mirror structures to demonstrate the proportionality of the reflection group delay and the stored energy. We used the well known matrix method to calculate the standing wave electromagnetic field inside the multilayer strucure. In all cases, normal incidence of light was assumed and the incident power spectral density was wavelength independent. In order to check whether the U/τ_{gr} ratio is independent from the wavelength, we calculated the maximum of this ratio within the wavelength range of high reflectivity ($|r|^2 > 0.997$) and then the relative difference from this maximum:

$$relvar(U/\tau_{gr}) = -\left(\max(U/\tau_{gr}) - U/\tau_{gr}\right)/\max(U/\tau_{gr}) \tag{7}$$

Because of (6), we expect that the change of $relvar(U/\tau_{gr})$ is much smaller than unity within the HR region.

First we investigated a simple quarter-wave (QW) stack with a reference vacuum wavelength of $\lambda_0 = 800$ nm comprising 10 pairs of low and high index layers: $S\,|\,(HL)^{10}\,|\,A$. The refracitve index of the substrate (S) is $n_s = 1.51$, the refractive index of air (A) is $n_A = 1.00$. H and L denote high and low refractive index layers of $\lambda_0/4$ optical thicknesses, respectively, with $n_H = 2.315$ and $n_L = 1.45$. The computed reflectance and $relvar(U/\tau_{gr})$ vs. wavelength functions are shown in Figure 1. We found that the realtive energy/group delay ratio is almost constant within the HR region, the maximum change in $relvar(U/\tau_{gr})$ being smaller than 0.005.

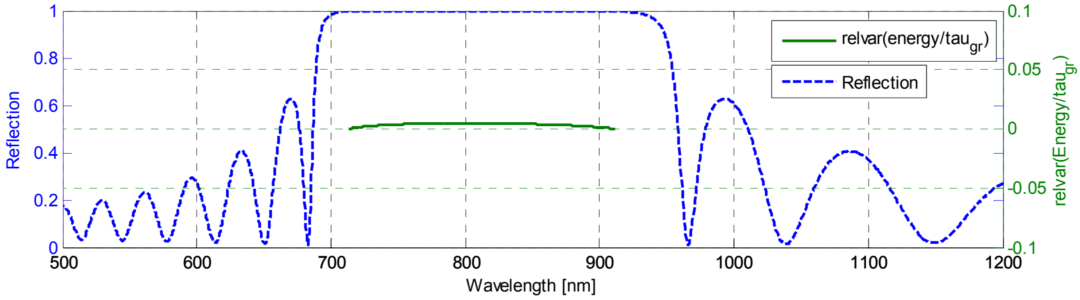

Fig. 1 QW stack. $\lambda_0 = 800$ nm, $n_s = 1.51$, $n_A = 1.00$, $n_H = 2.315$, $n_L = 1.45$

The second example is a dispersive, thin-film Gires-Tournois interferometer mirror having the following design: $S\,|\,(HL)^{10}\,H\,2L\,H\,L\,|\,A$. The reference wavelength and the refractive indices are the same as before. Here the maximum change in $relvar(U/\tau_{gr})$ is smaller than 0.0025.

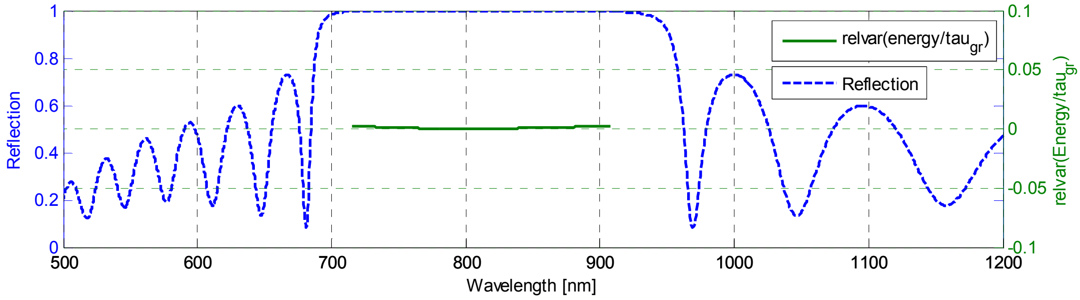

Fig. 2. GTI mirror. $\lambda_0 = 800$ nm, $n_s = 1.51$, $n_A = 1.00$, $n_H = 2.315$, $n_L = 1.45$

The third example is an ultrabroadband chirped mirror (UBCM) design, first reported and fully described in reference [6]. We note that this kind of UBCM-s are now used for dispersion control and feedback over an octave bandwidth. For this specific UBCM design, the maximum change in $relvar(U/\tau_{gr})$ is smaller than 0.015.

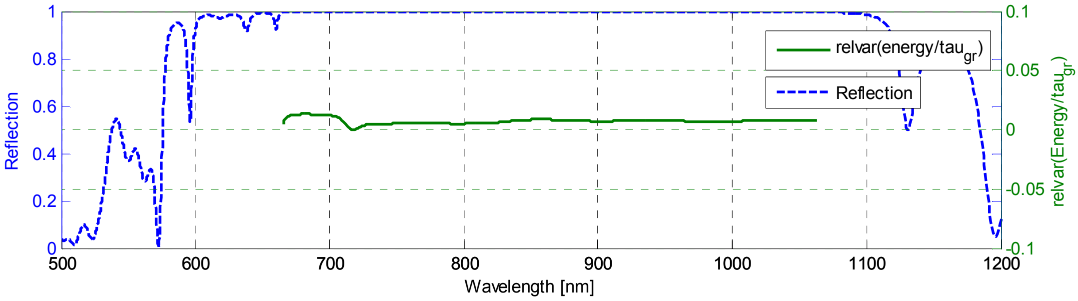

Fig. 3. UBCM mirror. $\lambda_0 = 790$ nm, $n_s = 1.51$, $n_A = 1.00$, $n_H = 2.315$, $n_L = 1.45$

The fourth structure we investigated is a multi-cavity Gires-Tournois interferometer (MCGTI) mirror, based on the design recently reported in Ref. [4]. Here $n_H = 2.026$, $n_L = 1.48$, $n_S = 1.51$, $n_A = 1.00$. This specific mirror design provides a huge negative GDD of -1200 fs^2 over a spectral bandwidth of ~50 nm, which can be explained by the extremely high energy stored in the multilayer structure. This fact may result in damage threshold problems when they are used in high power CPA systems. The maximum change in $relvar(U/\tau_{gr})$ is smaller than 0.015.

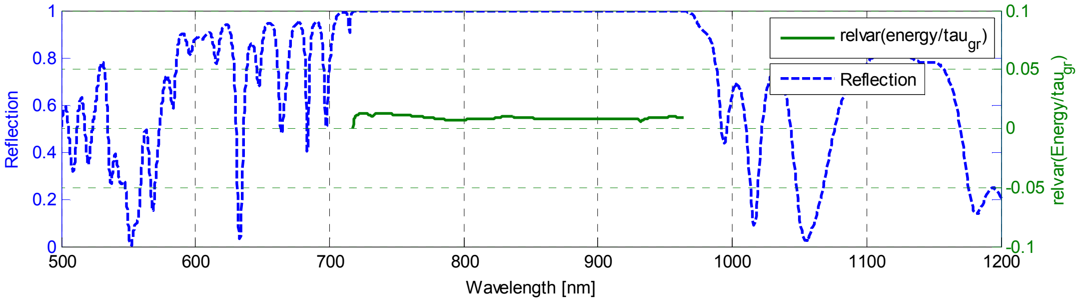

Fig. 4. MCGTI mirror. $\lambda_0 = 800$ nm, $n_s = 1.51$, $n_A = 1.00$, $n_H = 2.026$, $n_L = 1.48$

Based on the results listed above, we are convinced that the presented relation between group delay and energy storage in dispersive dielectric mirror coatings leads to a better understanding of dispersive dielectric mirror coatings and to higher performance, more stable optical coating designs.

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4. References

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